2020-2021 5

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8 5 40 .

1. $a \neq 0, a \in \mathbf{R}$ $y = 4ax^2$

A. $(a, 0)$ B. $(0, a)$ C. $(0, \frac{1}{16a})$ D. a

2. $C: y^2 = 2px(p > 0)$ F $M(2, m)$ $|MF| = 6$

C

A $y^2 = 2x$ B $y^2 = 4x$ C $y^2 = 8x$ D $y^2 = 16x$

3. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1(a > 0, b > 0)$ $y^2 = 4x$ $\sqrt{5}$

A $\frac{x^2}{5} - \frac{y^2}{4} = 1$ B $\frac{y^2}{5} - \frac{x^2}{4} = 1$ C $5x^2 - \frac{5y^2}{4} = 1$ D $5x^2 - \frac{4y^2}{5} = 1$

4. $x^2 + y^2 + 6x + 5 = 0$ $x^2 + y^2 - 6x - 91 = 0$

A B C D

5. F $y^2 = x$ $|AF| + |BF| = 3$ AB y

A $\frac{3}{4}$ B 1 C $\frac{5}{4}$ D $\frac{7}{4}$

6.

$x^2 + y^2 \leq 1$ $A(3, 0)$

$x + y = 4$

A $\sqrt{17} - 1$ B $\sqrt{17} - \sqrt{2}$ C $\sqrt{17}$ D $3 - \sqrt{2}$

7. F_1, F_2 $C: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1(a > 0, b > 0)$ O F_2 C

P $|PF_1| = \sqrt{6}|OP|$ C

A $\sqrt{5}$ B $\sqrt{3}$ C 2 D $\sqrt{2}$

8. $y^2 = 4x$ F $P(2, 0)$ AB

k_1, k_2 $\frac{k_1}{k_2} =$

A $-\frac{1}{2}$ B 2 C 1 D $\frac{1}{2}$

4 5 20 . .

5 0 3 .

9. xOy P $F_1\left(-\sqrt{3},0\right)$ $F_2\left(\sqrt{3},0\right)$ $\frac{1}{3}$ P

E E

A E $\frac{x^2}{3}-y^2=1(x\neq\pm\sqrt{3})$ B E $\sqrt{3}$

C E $\left(\quad\right)^2$ D $|AB|=2\sqrt{3}$ l 1

10. $C:y^2=2px\,(p>0)$ F l A C F $|FA|$ l

$\angle ABD=90^\circ$ $\triangle ABF$ $9\sqrt{3}$

A $|BF|=3$ B $\triangle ABF$

C F 3 D C $y^2=6x$

11. $ABCD$ BD

A AD BC 30° B AC BD 90°

C BC ACD $\frac{\sqrt{3}}{3}$ D ABC BCD $\sqrt{2}$

12. $y^2=2px\,(p>0)$ F l C

$|BC|=2|BF|$ $|AF|=3$

A $y^2=9x$ B $y^2=6x$ C $y^2=3x$ D $y^2=\sqrt{3}x$

4 5 20 2 3 .

13. l $M(1,1)$ $\frac{x^2}{4}+\frac{y^2}{3}=1$ AB M

l _____.

14. $ABCD$ $\angle ADC=90^\circ$ $\angle A=45^\circ$ $AB=2$ $BD=5$.

$\cos\angle ADB=$_____ [redacted] $BC=$_____.

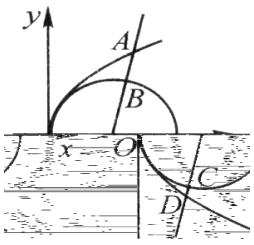
15. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\left(a>b>0\right)$ $\frac{x^2}{m^2}-\frac{y^2}{n^2}=1(m>0,n>0)$ F_1,F_2

P e_2

$\angle F_1PF_2=\frac{\pi}{3}$ $e_1^2+e_2^2$ _____.

16. $y^2=4x$ $(x-1)^2+y^2=1$

A,B,C,D $|\overrightarrow{AB}|\cdot|\overrightarrow{CD}|=$_____



6 70 17 10

12

17. 10 ABC A,B,C $(\sin B-\sin C)^2=\sin^2 A-\sin B\sin C$

A

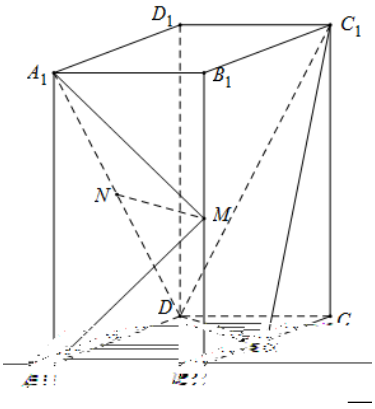
$\sqrt{2}a+b=2c$ $\sin C$

18. 12 $ABCD-A_1B_1C_1D_1$

$AA_1=4$ $AB=2$ $\angle BAD=60^\circ$ BC,BB_1,A_1D

$MN//$ C_1DE

$A-MA_1-N$



19. 12 $C:\frac{x^2}{b^2}+\frac{y^2}{a^2}=1(a>b>0)$ $\frac{\sqrt{3}}{2}$ C 4

C

$l:y=kx+1$ C k AB

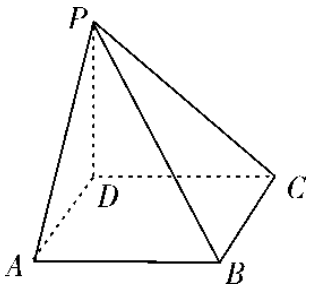
O AB

20. 12 $P-ABCD$ $PD\perp$ $ABCD$

PAD PBC l

$l\perp$ PDC

$PD=AD=1$ Q l PB QCD



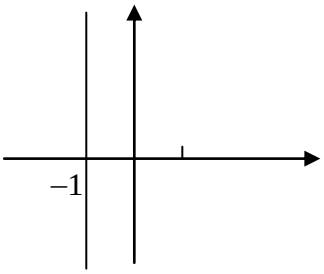
21. 12 $F(1\,0)$ $l:x=-1$ P

P l Q $\overrightarrow{QP}\cdot\overrightarrow{QF}=\overrightarrow{FP}\cdot\overrightarrow{FQ}$

P C

F C l M

$\overrightarrow{MA}=\lambda_1\overrightarrow{AF}$ $\overrightarrow{MB}=\lambda_2\overrightarrow{BF}$ $\lambda_1+\lambda_2$.



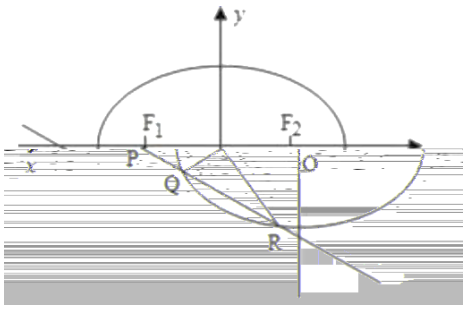
22. 12 $C:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$ F_1,F_2

$\frac{\sqrt{2}}{2}$ $y=x+\sqrt{2}$.

C

$k(k\neq 0)$ l x C

$\angle RF_1F_2=\angle PF_1Q$ l k .



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1-4 CDCD 5-8 CABD

8. $F(1,0)$, $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$, $D(x_4,y_4)$,

$AB: y=k_1(x-2)$ $AF: y=\dfrac{y_1}{x_1-1}(x-1)$ $BF: y=\dfrac{y_2}{x_2-1}(x-1)$,

$$\begin{cases} y=\dfrac{y_1}{x_1-1}(x-1) \\ y^2=4x \end{cases} \quad x^2-2x-\dfrac{4(x_1-1)^2}{y_1^2}x+1=0$$

$x_1x_3=1$, $x_3=\dfrac{1}{x_1}$, $y_1=\dfrac{y_1}{x_1-1}\left(\dfrac{1}{x_1}-1\right)=-\dfrac{y_1}{x_1}$ $x_4=\dfrac{1}{x_2}$, $y_4=-\dfrac{y_2}{x_2}$

$$k_2=\frac{\dfrac{-y_1}{x_1}-\left(-\dfrac{y_2}{x_2}\right)}{\dfrac{1}{x_1}-\dfrac{1}{x_2}}=\frac{-x_2y_1+x_1y_2}{x_2-x_1}=\frac{-x_2\left[k_1(x_1-2)\right]+x_1\left[k_1(x_2-2)\right]}{x_2-x_1}$$

$$=\frac{-k_1x_1x_2+2k_1x_2+k_1x_1x_2-2k_1x_1}{x_2-x_1}=\frac{2k_1(x_2-x_1)}{x_2-x_1}=2k_1$$

$$\frac{k_1}{k_2}=\frac{1}{2}$$

4 5 20 . .

5 0 3 .

9. AC 10. BCD 11. BD 12. AC

4 5 20 2 3 .

13. $3x+4y-7=0$ 14. $\dfrac{\sqrt{23}}{5}$ 5 15. $\dfrac{2+\sqrt{3}}{2}$ 16. 1

6 70 17 10

12

17. $\sin^2B+\sin^2C-\sin^2A=\sin B\sin C$ $b^2+c^2-a^2=bc$ 2

$\cos A=\dfrac{b^2+c^2-a^2}{2bc}=\dfrac{1}{2}$ 4

$0^\circ<A<180^\circ$ $A=60^\circ$ 5

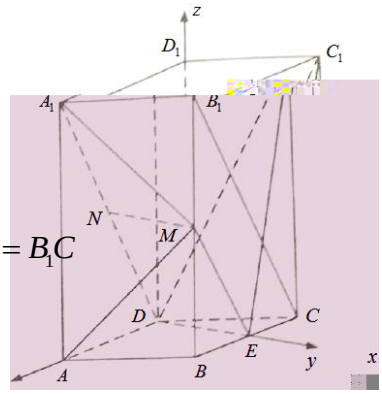
$B=120^\circ-C$ $\sqrt{2}\sin A+\sin\left(120^\circ-C\right)=2\sin C$ 6

$\dfrac{\sqrt{6}}{2}+\dfrac{\sqrt{3}}{2}\cos C+\dfrac{1}{2}\sin C=2\sin C$ $\cos\left(C+60^\circ\right)=-\dfrac{\sqrt{2}}{2}$ 8

$0^\circ<C<120^\circ$ $\sin\left(C+60^\circ\right)=\dfrac{\sqrt{2}}{2}$ 9

$\sin C=\sin\left(C+60^\circ-60^\circ\right)=\sin\left(C+60^\circ\right)\cos60^\circ-\cos\left(C+60^\circ\right)\sin60^\circ=\dfrac{\sqrt{6}+\sqrt{2}}{4}$ 10

18. B_1C, ME, BB_1, BC



$ME//B_1C$ $ME=\frac{1}{2}B_1C$ 1

N A_1D $ND=\frac{1}{2}A_1D$ 2

$A_1B_1//DC$ $A_1B_1=DC$ $A_1D//B_1C$ $A_1D=B_1C$

$ME//ND$ $ME=ND$ 3

$MNDE$ $MN//ED$ 4

$MN\not\subset$ EDC_1, $ED\subset$ EDC_1

$MN//$ EDC_1 5

$DE\perp DA$, D $\overrightarrow{DA}$ x $D-xyz$

$A(2,0,0)$ $A_1(2,0,4)$ $M(1,\sqrt{3},2)$ $N(1,0,2)$ 6

$\overrightarrow{A_1A}=(0,0,-4)$ $\overrightarrow{A_1M}=(-1,\sqrt{3},-2)$ $\overrightarrow{A_1N}=(-1,0,-2)$ $\overrightarrow{MN}=(0,-\sqrt{3},0)$

$\vec{m}=(x,y,z)$ A_1MA $\begin{cases}\vec{m}\cdot\overrightarrow{A_1M}=0\\ \vec{m}\cdot\overrightarrow{A_1A}=0\end{cases}$

$\begin{cases}-x+\sqrt{3}y-2z=0\\ -4z=0\end{cases}$ $\vec{m}=(\sqrt{3},1,0)$ 8

$\vec{n}=(p,q,r)$ A_1MN $\begin{cases}\vec{n}\cdot\overrightarrow{MN}=0\\ \vec{n}\cdot\overrightarrow{A_1N}=0\end{cases}$

$\begin{cases}-\sqrt{3}q=0\\ -p-2r=0\end{cases}$ $\vec{n}=(2,0,-1)$ 10

$\cos\langle\vec{m},\vec{n}\rangle=\frac{\vec{m}\cdot\vec{n}}{|\vec{m}\ \ \vec{n}|}=\frac{2\sqrt{3}}{2\times\sqrt{5}}=\frac{\sqrt{15}}{5}$ 11

$A-MA_1-N$ $\frac{\sqrt{10}}{5}$ 12

19. c $\begin{cases}a=2\\ \frac{c}{a}=\frac{\sqrt{3}}{2}\end{cases}$ $\begin{cases}a=2\\ c=\sqrt{3}\end{cases}$ 3

$b^2=a^2-c^2=4-3=1$ 4

C $\frac{y^2}{4}+x^2=1$ 5

$A(x_1,y_1)$ $B(x_2,y_2)$ l $y=kx+1$ $\frac{y^2}{4}+x^2=1$

$(k^2+4)x^2+2kx-3=0$ * 6

$x_1+x_2=-\frac{2k}{k^2+4}$ $x_1x_2=-\frac{3}{k^2+4}$ 7

AB O

$\overrightarrow{OA}\cdot\overrightarrow{OB}=0$ $x_1x_2+y_1y_2=0$ 8

$y_1y_2=k^2x_1x_2+k(x_1+x_2)+1$

$-\frac{3(k^2+1)}{k^2+4}-\frac{2k^2}{k^2+4}+1=0$ $k=\pm\frac{1}{2}$ 10

* 0

$k=\pm\frac{1}{2}$ AB O

$x_1x_2=\pm\frac{4}{17}, x_1x_2=-\frac{12}{17}$ $|AB|=\frac{4\sqrt{65}}{17}$ 12

20. $ABCD$ $AD//BC$

$AD\not\subset$ PBC $BC\subset$ PBC $AD//$ PBC 1

$AD\subset$ PAD $PAD\cap$ $PBC=l$ $AD//l$ 2

$P-ABCD$ $ABCD$

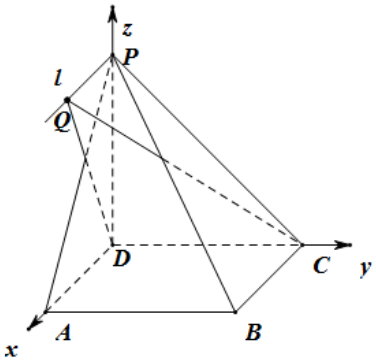
$AD\perp DC,\therefore l\perp DC,$ 3

$PD\perp$ $ABCD$ $AD\perp PD,\therefore l\perp PD,$ 4

$CD\cap PD=D$ $l\perp$ PDC 5

$D-xyz$ $PD=AD=1$

$D(0,0,0), C(0,1,0), A(1,0,0), P(0,0,1), B(1,1,0)$ 6



$Q(m,0,1)$ $\overrightarrow{DC}=(0,1,0), \overrightarrow{DQ}=(m,0,1), \overrightarrow{PB}=(1,1,-1)$

QCD $\vec{n}=(x,y,z)$ $\begin{cases}\overrightarrow{DC}\cdot\vec{n}=0\\ \overrightarrow{DQ}\cdot\vec{n}=0\end{cases}$ $\begin{cases}y=0\\ mx+z=0\end{cases}$

$x=1$ $z=-m$ QCD $\vec{n}=(1,0,-m)$ 8

$\cos<\vec{n},\overrightarrow{PB}>=\frac{\vec{n}\cdot\overrightarrow{PB}}{|\vec{n}||\overrightarrow{PB}|}=\frac{1+0+m}{\sqrt{3}\cdot\sqrt{m^2+1}}$ 9

$|\cos<\vec{n},\overrightarrow{PB}>|=\frac{|1+m|}{\sqrt{3}\cdot\sqrt{m^2+1}}=\frac{\sqrt{3}}{3}\cdot\sqrt{\frac{1+2m+m^2}{m^2+1}}$ 10

$=\frac{\sqrt{3}}{3}\cdot\sqrt{1+\frac{2m}{m^2+1}}\le\frac{\sqrt{3}}{3}\cdot\sqrt{1+\frac{2|m|}{m^2+1}}\le\frac{\sqrt{3}}{3}\cdot\sqrt{1+1}=\frac{\sqrt{6}}{3}$ 11

$m=1$

PB QCD $\frac{\sqrt{6}}{3}$. 12

21. $P(x,y)$ $Q(-1,y)$ 1

$\overrightarrow{QP}\cdot\overrightarrow{QF}=\overrightarrow{FP}\cdot\overrightarrow{FQ}$ $(x+1,0)\cdot(2,-y)=(x-1,y)\cdot(-2,y)$ 3

$C: y^2=4x.$ 4

AB $x=my+1(m\neq0)$ $A(x_1,y_1)$ $B(x_2,y_2)$

$M(-1,-\frac{2}{m})$ 5

$\begin{cases}y^2=4x\\ x=my+1\end{cases}$ $y^2-4my-4=0$ 6

$\Delta = 16m^2 + 12 > 0$ $\begin{cases} y_1 + y_2 = 4m \\ y_1 y_2 = -4 \end{cases}$ 8

$\overrightarrow{MA} = \lambda_1 \overrightarrow{AF}$ $\overrightarrow{MB} = \lambda_2 \overrightarrow{BF}$ $y_1 + \frac{2}{m} = -\lambda_1 y_1$ $y_2 + \frac{2}{m} = -\lambda_2 y_2$ 9

$\lambda_1 = -1 - \frac{2}{my_1}$ $\lambda_2 = -1 - \frac{2}{my_2}$ 10

$\lambda_1 + \lambda_2 = -2 - \frac{2}{m}(\frac{1}{y_1} + \frac{1}{y_2}) = -2 - \frac{2}{m} \cdot \frac{y_1 + y_2}{y_1 y_2} = -2 - \frac{2}{m} \cdot \frac{4m}{-4} = 0$ 12

22. $F_1(-c,0), F_2(c,0)$ $\frac{\sqrt{2}}{2}$ $\frac{c}{a} = \frac{\sqrt{2}}{2}$

$a = \sqrt{2}c$ $b = \sqrt{a^2 - c^2} = c$ 1

$x^2 + y^2 = b^2$

$y = x + \sqrt{2}$ $\frac{|\sqrt{2}|}{\sqrt{2}} = 1 = b$ 3

$a = \sqrt{2}$

C $\frac{x^2}{2} + y^2 = 1$ 4

$Q(x_1, y_1), R(x_2, y_2), F_1(-1,0)$

$\angle RF_1F_2 = \angle PF_1Q$ QF_1 RF_1 x

$k_{QF_1} + k_{RF_1} = 0$ $\frac{y_1}{x_1 + 1} + \frac{y_2}{x_2 + 1} = 0$ 5

$x_1y_2 + y_2 + x_2y_1 + y_1 = 0$ 6

$PQ: y = kx + t$ $(1 + 2k^2)x^2 + 4ktx + 2t^2 - 2 = 0$

$\Delta = 16k^2t^2 - 4(1 + 2k^2)(2t^2 - 2) > 0$ $t^2 - 2k^2 < 1$ 7

$x_1 + x_2 = \frac{-4kt}{1 + 2k^2}, x_1x_2 = \frac{2t^2 - 2}{1 + 2k^2}$ 8

$y_1 = kx_1 + t, y_2 = kx_2 + t$ $(k + t)(x_1 + x_2) + 2t + 2kx_1x_2 = 0$

$t = 2k$ 9

l $y = kx + 2k$ $y = k(x + 2)$ l $(-2,0)$ 10

$t = 2k$ $2k^2 < 1$ $-\frac{\sqrt{2}}{2} < k < 0$ $0 < k < \frac{\sqrt{2}}{2}$

l k $\left(-\frac{\sqrt{2}}{2}, 0\right) \cup \left(0, \frac{\sqrt{2}}{2}\right)$ 12